Utilizing Topographic Finance to Understand Volatility

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ABSTRACT

Visual representation methods are a common problem in econometrics and finance in order to describe system dynamics. In this paper we address this problem by using the bi-harmonic oscillation process and the Brownian motion components, to generate a three-dimensional volatility surface.

The empirical analysis have been carried out on the S&P500 Index, the 10-year US Treasury Rates, and the West Texas Intermediate oil price, by using 85 daily closing observations, at the purpose to show how visualization of volatility can help understand longitudinal movements of price within different asset classes.

1. INTRODUCTION

Topographic finance is the study of surfaces to describe financial systems in multiple dimensions. We naturally see the world in three-dimensions and four-dimensions, whereby four-dimension space is the Cartesian system with time. A problem in econometrics and finance is the visual representation methods used to see the system dynamics. For example, in financial markets a price curve has time and price; but what is actually governing the price dynamic? Some have proposed that price dynamics are behavioral and do not exhibit a rational maximization of a utility function (Mandelbrot and Hudson, 2004). Since financial markets are seemingly void of rationality we need to understand the fear generated due to the inherited uncertainty. Volatility is related to the variance of returns, but there is a lot to be understood

from the vector–space of volatility. Understanding the magnitude, direction, and emerging dynamics will help economists and financial market participants better forecast a particular financial system.

In this paper, we set out to show how to graph volatility and interpret the price dynamics of equity, bond, and oil markets. We will exploit the use of a bi-harmonic oscillation process and the components of a Brownian motion system to generate a three-dimensional volatility surface. The volatility surfaces were generated with daily closing prices on the S&P500 Index, 10-year US treasuries, and West Texas Intermediate oil spot price from November 10, 2014 to February 27, 2015. The period was selected to provide a mid-range surface window, (i.e. between 60-day to 90-day window). These surfaces are described with time, drift, and volatility.

The paper is organized as follows: in the next section, we will give an overview about the literature in the field of investor behavior, Section 3 presents the longitudinal descriptive research design, and Section 4 will be devoted to the used data. In Section 5 we will apply the described framework to three financial time series. Finally, Section 6 will be devoted to conclusions.

2. LITERATURE REVIEW

In terms of contemporary theories on investor behavior, reflexivity is one such theory that might be classified in the behavioral finance category. Reflexivity is loosely defined as a theory describing a feedback mechanism on a particular system, whereby certain conditions might magnify effects or diminish them. Soros (2003) proposed eight variables to model the currency market utilizing variables and arrows to understand the changing dynamics (p. 75).

Unfortunately the Soros's reflexivity model does not lend itself to an easy description in terms of volatility.

Thaler (1993) showed that irrational investors can be framed as a source of risk by defining the amount of holdings of an asset for a rational and irrational investor (pp. 28-28). See equation 1 and 2 for the rational and irrational holding amounts respectively.

$$\lambda_t^R = \frac{r+p_{t+1}-(1+r)p_t}{2\gamma(\sigma_{p_{t+1}}^2)} \qquad (1)$$

$$\lambda_t^I = \frac{r+p_{t+1}-(1+r)p_t}{2\gamma(\sigma_{p_{t+1}}^2)} + \frac{p_t}{2\gamma(\sigma_{p_{t+1}}^2)} \qquad (2)$$

R and *I* represent rational and irrational investors, *t* is the time period, *r* equals rate of return, p_t is price at time *t*, γ represents the coefficient of absolute risk aversion, and σ^2 pertains to the variance of the asset.

To define the price rule of an asset, only exogenous parameters and the public information of irrational trader's misperception are needed (p. 30). See equation 3 for the pricing rule of an asset proposed by Thaler. Fortunately Thaler's equations do make use of variance variables.

$$p_t = 1 + \frac{\mu\,(p_t - p^*)}{1+r} + \frac{\mu p^*}{r} - \frac{(2\gamma)\mu^2\sigma_p^2}{r(1+r)^2} \qquad (3)$$

The variable μ represents the percentage of irrational traders in the model and 1-μ equals the number of rational traders. The term p^* is emblematic of average bullishness of irrational traders.

Efficient market theory predicts that there is no advantage of special information; therefore any abnormal profits are quickly taken by arbitrageurs. When mispricing is suspected it is very likely that a continued increase or decrease of price will ensue due to the limiting number of rational traders entering the market, this seems to nullify the theory of efficient markets (Shiller, 2005, p. 181). The dynamics Shiller describes is due to the limited number of rational traders to offset the herd effect that adds to the volatility in asset prices. By understanding the herd mentality or reflexivity of the market we can build better forecasting methods utilizing the evolution of the volatility function.

Black and Scholes (1973) showed that options can be modeled using a Brownian motion system, whereby drift and volatility parameters are part of the Black–Scholes model. What is interesting from a visualization perspective is the use of the drift and volatility parameters within the Black–Scholes model for two of the dimensions of a volatility surface—all we need to add is time. Equation 3 represents a Brownian motion system for the stock price process.

$$dS(t) = \mu S(t) + \sigma S(t) W(t) \qquad (3)$$

The parameter μ is the drift, $S(t)$ is the price at time t, σ is the volatility, and $W(t)$ represents the white noise stochastic process.

By using a bi-harmonic (v4) spline interpolation we can develop a state-space where a set of three-dimensional points can be fitted to a surface ("Curve Fitting Toolbox," n.d.). The spline interpolation allows for a smooth transition between points, which is easier for visualization and interpretation purposes. Due to the nature of volatility, a harmonic interpolation method seems to allow a graphing of potential regions of drift and volatility for near future time periods, which is similar to a wave function of a particle before the wave function collapses through measuring a certain property of that particle. Cottrell (2015) showed

that by understanding the probabilities of the state-space we can improve on the forecasting of volatility near term when using a receding horizontal control and stochastic programming method.

3. METHODOLOGY

3.1 Research Design and Rationale

The strategy of inquiry for this paper utilized a longitudinal descriptive research design, in which we took the volatility of different markets and their respective drift characteristics throughout time. The purpose of this study was to show that visualization of volatility can help understand longitudinal movements of price within different asset classes.

The independent variables are drift and time, whereas the dependent variable is the volatility. The S&P 500, West Texas Intermediate (WTI), and 10-year Treasury were surfaced using the bi-hamonic interpolation method assuming the independent and dependent variables.

3.2 Instrumentation

The instrument used in this study was a volatility surface in three dimensions, but utilizing drift and time to explain volatility. Standard volatility surfacing incorporates volatility, time to maturity, and strike price of an option. We wanted to show that through this instrument a researcher or a trader can analyze and dissect the characteristics of any asset class without being dependent on an option market. Reliability of this instrument was shown to perform similar visualization descriptions for the equity, bond, and energy markets.

3.3 Operational definitions

We defined volatility to be the related to variance of the log return of the S&P 500, WTI, or 10-year Treasury. The drift variable defines the direction and magnitude of the momentum of the price curve. Our time variable is set at one day intervals, but this method can be applied at larger or smaller time scales.

3.4 Threat to validity

The internal validity is maintained due to the measuring device we are using. Volatility surface modeling has been well established in the option markets to understand volatility and time decay dynamics. This study examines how a volatility surface can be used as an instrument to analyze pricing dynamics outside of the option market. The external validity is considered well established because we show how to utilize topographic finance in multiple markets, such as equity, bonds, and energy. However, to improve on the external validity of this study more research needs to be conducted in different equity, bond, and energy markets.

4. THE DATA

4.1 Population

For the purposes of this study, we gathered the time series of daily closings from November 3rd, 2014 to February 27th, 2015 for the S&P500 Index[1] level, the 10-Year US Treasury Constant Maturity Rate[2], and the West Texas Intermediate (WTI) Oil Price per barrel, from the Federal Reserve Economic Data (FRED) database, which is provided and maintained by the Research division of Fed of St. Louis. The FRED database allows for the downloading, graphing and tracking of roughly 279,000 time series from 79 sources about banking, consumer

[1] The S&P500 Index is a measure of the large cap U.S. equity market, and includes the 500 U.S. companies with the larger market capitalization, publicly listed on NYSE, AMEX, and NASDAQ. It this is a price index, thus it does not consider any dividend.
[2] Yields on actively traded non-inflation-indexed issues adjusted to constant maturities.

price indexes, employment and populations, exchange rates, GDP, interest rates, monetary aggregates, producer price indexes, reserves and monetary base, and U.S. financial data.

4.2 Sampling and Sampling Procedure

Thus, we have a daily time series with 85 observations for each considered item. Then, for each time series, we calculated the daily log-returns, the 5-day window volatility, and the 5-day window drift. For the days with missing data (6 for the 10-year rates, and 5 for the S&P500 Index and WTI Oil price), we interpolated them with the arithmetic average between the data of the day before, and the one-day after.

4.3 Procedure for surfacing

In order to get the volatility surface for the S&P 500 Index, the 10-year Treasury Rates and the WTI prices, as described in Section 2 we used the MATLAB Curve Fitting Tool, putting the time on the x-axis, the drift on the y-axis, and finally the volatility on the z-axis. Each of the considered items, have been imported separately for ease of MATLAB usage. Hence, we obtained three volatility surfaces through interpolation using the bi-harmonic (4v) method. For the surface, only 80 observations are used because of the 5-day window implemented for calculating volatility and drift—the first 5 observations were removed before the surface fitting due to null values for volatility and drift.

Hereby, in order to have a clearer idea, about the used time series, we provide a table with the descriptive statistics for the S&P500 Index, 10-year Treasury Rates and WTI oil price per barrel:

	Max	Min	Mean	Median	St.Dev	Variance	Kurtosis	Skewness
10-y Treas	0,064	-0,056	-0,002	-0,004	0,024	0,001	0,123	0,207
WTI	0,080	-0,091	-0,005	-0,009	0,032	0,001	0,191	0,265
S&P500	0,024	-0,018	0,001	0,000	0,008	0,000	0,894	0,230

5. RESULTS

As has been shown, we have used a bi-harmonic interpolation surfacing method with our data on the S&P500 Index, 10-year Treasury, and WTI between November 10, 2014 and February 27, 2015. We modeled the 5-day volatility and drift with time. The surface window was fitted with 80 days of data. The concept of a surface window is to understand the volatility from different time perspectives, similar to the standard practice to reference different price curves within different time intervals. Traders reference price curves from a day or weekly perspective, as well as from an hour or 30-minute vantage point. These different perspectives allow for the trader or researcher to understand support and resistance levels for a particular asset. Utilizing different surface windows allows for an understanding of different volatility characteristics. This paper was not meant to delve into the ideal surface window or the ideal moving window for the volatility and drift variables, but to convey that such customizations can be developed to meet the needs of different traders and researchers.

5.1 S&P500 Index

The qualitative characteristics of the S&P500 Index volatility surface are shown in Figure 1. As is shown in Figure 1, the volatility is heteroscedastic with two distractive volatility episodes—around the middle of the investigated timeframe. Being characteristic of heteroscedastic volatility, the surface in Figure 1 is relatively calm for most of the dataset but peaks in short intervals—represented in the early part and late part of the time series. The

surface can be considered the state-space of possible volatility and drift values relative to time, assuming that the state-space can be fitted using a bi-harmonic spline interpolation method. The black dots on the surface are the actual data points of the time series.

Before the closing of the day, the possible closing price is not known; therefore the state-space can be considered a wave function. Once the closing price is established the wave function collapses and the surface refits to accommodate the new information. A trader or researcher can consider the volatility surface a historical representation of volatility. Since volatility is heteroscedatic, low volatility over a certain time will beget higher volatility. Higher volatility is short lived and will settle down to normal levels within the time series.

For example, we see that there are many points in the low volatility range in the newest closing prices of the S&P500 Index. This will lead to the qualitative reading of the surface that the next time period should be in lower volatility but that extended time in this range will lead to a spike in volatility. The drift is also slightly biased negative. Therefore the next data point, using a qualitative analysis, would be predicted to be a negative drift with slight volatility. To determine the actual probability of continued regime or regime switching, probability curves of the surface window need to be analyzed.

Figure 2 and 3 show the PDF and CDF curves for the S&P500 Index for our dataset. As expected, volatility was skewed. The mean and one-sigma of volatility for the S&P500 Index within our investigated timeframe was .006 and .004 respectively when using a beta curve fitting. One possible way to utilize the PDF curve for trading is to determine if current volatility is below or above the surface window average. If the current volatility is below the mean then a trader should assume an increased volatility is on the horizon, albeit a heuristic that is a rough

estimate of the movement of volatility. A better approach is to analyze the CDF curve and determine the cumulative probability of different volatility ranges. For example, Figure 3 shows that there is a 50% probability that the volatility will about $.5x10^{-3}$ or less— a relatively low range in the dataset. If we look at the cumulative probability at the 80% range, volatility was predicted at $10x10^{-3}$ or less. Volatility is assumed to increase using the 80% range when compared to the low current volatility points, but reading volatility can be idiosyncratic. Let's remind that volatility will spike usually after a period of sustained lower volatility. If we have a sustained period of lower volatility of $.8x10^{-4}$ in the surface plot and you compare it to the last 20% (80% to 100%) of the CDF curve it is possible to find that there is an 80% probability of higher than $10x10^{-3}$ for volatility. The trick to reading CDF curves for volatility analysis is to invert the probability when we are in a trough and use the standard reading of the CDF curve when at a high point in volatility, but let us keep in mind that there is a decaying function for this inverted probability for low volatility. As more time lapses the probability of higher volatility increases for low volatility epochs, resulting in a standard reading of the CDF curve at the tail-end of the low volatility epoch.

A good rule to follow is to take the current volatility reading when it is relatively high and retrieve the cumulative probability. Cumulative probability is the probability that the current value will be lower the next time period. For points relatively low in volatility, it is possible to retrieve the cumulative probability and determine that there is *x*% chance of higher volatility, but only when there has been sustained relatively lower volatility within the surface window. To understand the time duration probability of low volatility epochs we need to do further statistical analysis not shown in this paper.

To read the CDF curve of the drift, a trader or a researcher can just take the current 5-day drift and compare it on the CDF curve to determine the probability of lower trend. Reading drift is a relatively straight forward process compared to the idiosyncrasy of volatility.

In the final analysis, the volatility of the S&P500 Index on February 27, 2015 was relative low compared to the volatility episodes in the middle of the time series. With such low volatility the CDF curve suggests that future volatility will have a higher probability to increase. Remember that increased volatility means a change in drift or a change in returns of a position in the asset. This is a good point to place a short or long in the market relative to an 80-day surface window. To determine if the market should be longed or shorted we need to look at similar CDF curves of drift to determine what the cumulative probability of higher drift would be. A high probability of positive drift will lead to the decision to long the market when coupled with lower current volatility. When negative drift is more probable with lower current volatility a trader should short the market. Based on the data presented for the S&P500 Index the signals suggest shorting the market for the near term.

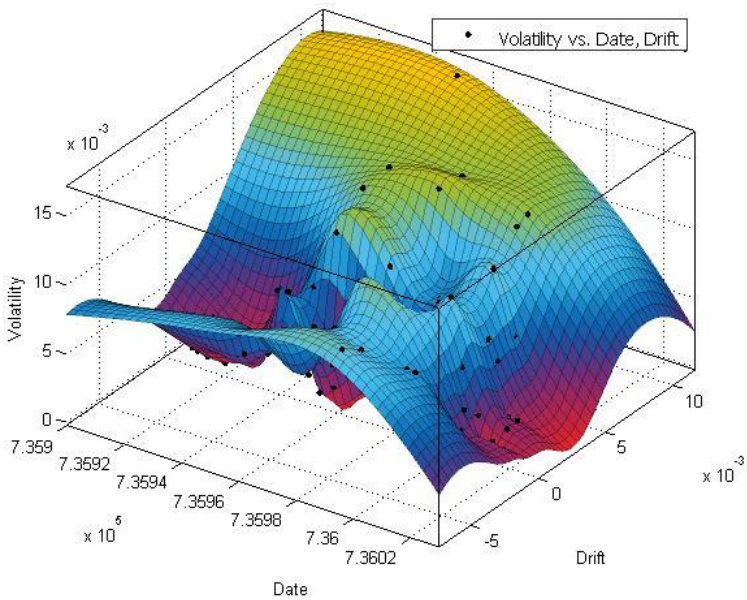
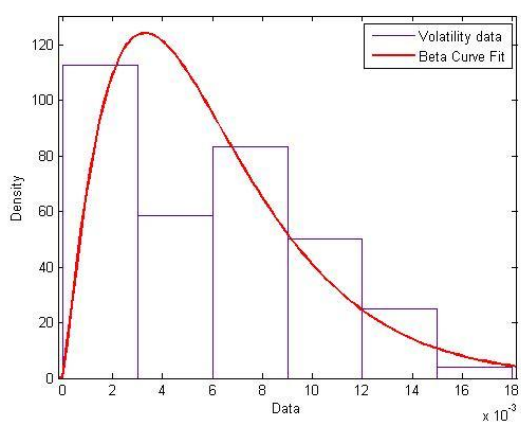


Figure 1. S&P500 volatility surface.



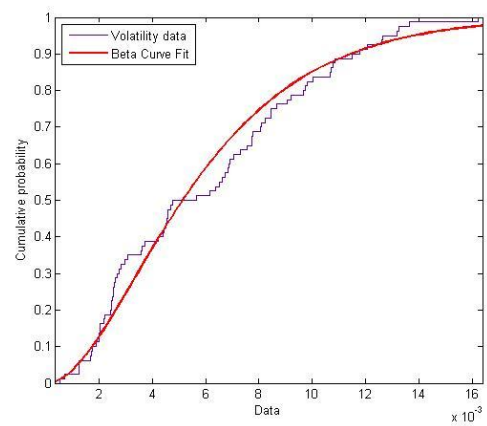


Figure 2. S&P500 PDF of volatility.

Figure 3. S&P500 CDF of volatility.

5.2 10-Year Treasury

The volatility surface of the 10-Year Treasury Rates exhibited similar peaking as the S&P500 Index during the investigated time series. See Figure 4 on the volatility surface for the 10-Year Treasury Rates. The surface interpolation does seem to suggest a slightly more

volatility than the interpolation of the S&P500 Index for the same time period, (e.g. last week in February). A quick look at the drift characteristics in Figure 4 suggests a positive movement leading to higher yields.

Figure 5 and 6 show the PDF and CDF curves for the 10-Year Treasury Rates for the investigated time series. In Figure 5 it is obvious that the distribution is slightly different than in Figure 2—the 10-year Treasury Rates shows a near normal distribution compared to the S&P500 Index from November 10, 2014 to February 27, 2015. The mean and one-sigma of volatility for the 10-year Treasury within our investigated timeframe was .020 and .009 respectively using a normal curve fitting. The CDF curve in Figure 6 does seem to fit better than the CDF curve in Figure 3, most likely due to the near normal distribution for volatility. The 80% cumulative probability for the 10-year Treasury Rates was .025 for volatility.

In the final analysis, the volatility of the 10-year Treasury Rates on February 27, 2015 was relatively mid-ranged compared to the volatility episodes in the middle of the time series. With such current volatility the CDF curve suggests that future volatility will have a higher probability to decrease near term. Based on the data presented for the10-year Treasury the signals suggest that yields will increase in the near term.

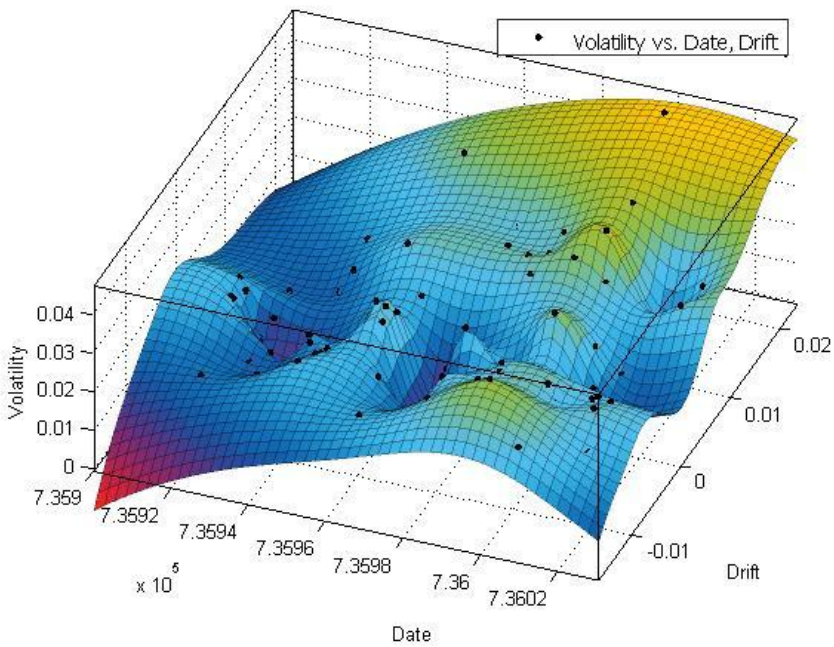
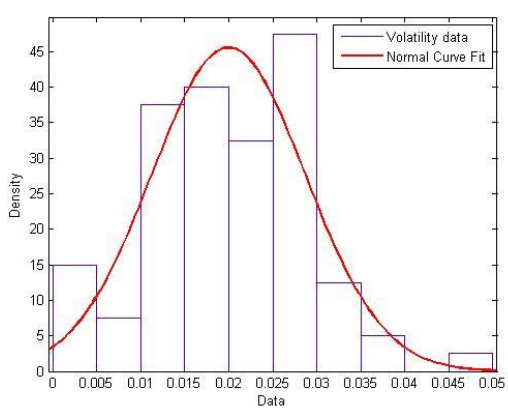


Figure 4. 10-Year Treasury volatility surface.



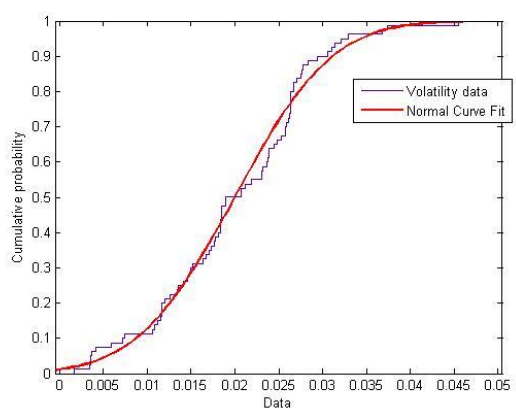


Figure 5. 10-Year Treasury PDF of volatility. *Figure* 6. 10-Year Treasury CDF of volatility.

5.3 WTI

The volatility surface of the WTI oil contract exhibited much different peaking compared to the S&P500 Index during the investigated time series. See Figure 7 on the volatility surface

for the WTI. The surface topology for the last week of data suggest that a volatility peak has taken place and that volatility will relatively reduce compared to that volatility peaking. A quick look at the drift characteristics in Figure 7 suggests a positive movement in the near term.

Figure 8 and 9 show the PDF and CDF curves for the WTI for the investigated time series. In Figure 8 it is obvious that the distribution is different than in Figure 2—the WTI shows a near normal distribution compared to the S&P500 Index from November 10, 2014 to February 27, 2015. The mean and one-sigma of volatility for the WTI within our investigated timeframe was .026 and .012 respectively using a normal curve fitting. The CDF curve in Figure 9 does seem to fit better than the CDF curve in Figure 3, but worse compared to Figure 6. The better fitting CDF curve fitting for the WTI compared to the S&P500 Index is due to the near normal distribution of volatility. The 80% cumulative probability for the WTI was round .04 for volatility.

In the final analysis, the volatility of the WTI on February 27, 2015 was relatively elevated compared to the higher volatility episodes of the time series. With such current volatility, the CDF curve suggests that future volatility will have a higher probability to decrease near term. Based on the data presented for the WTI, the signals suggest a short-term long position, due to less expected volatility and slight positive drift.

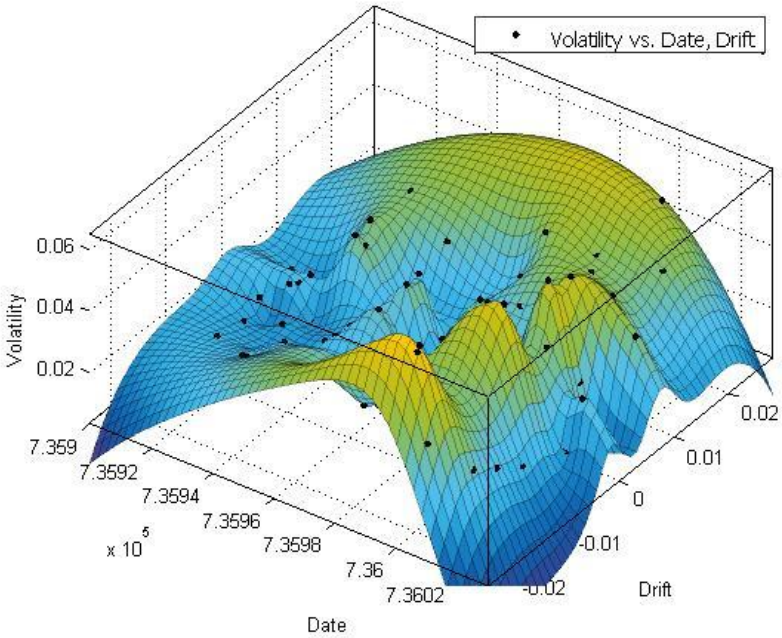
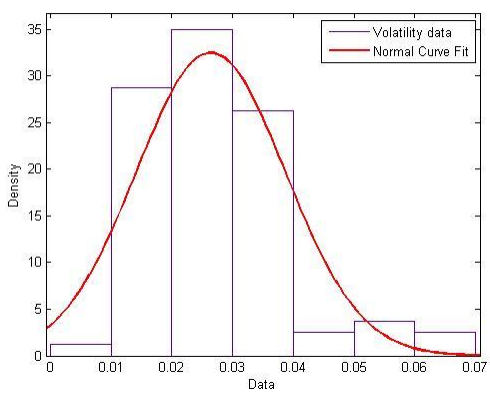


Figure 7. WTI volatility surface.



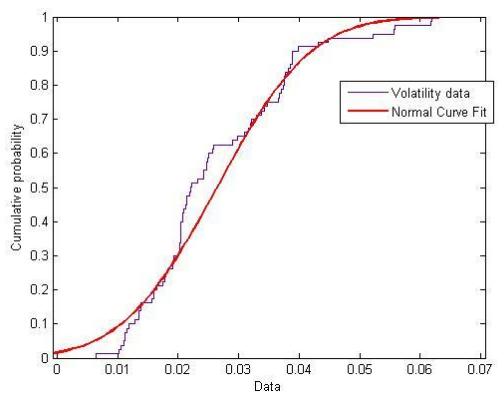


Figure 8. WTI PDF of volatility.

Figure 9. WTI Treasury CDF of volatility.

In summary, a trader can utilize the surface techniques described here by looking at the current volatility and drift, as well as the PDF and CDF curves to determine the probability of volatility movements. Based on this analysis, the S&P500 Index should be shorted, the 10-year

Treasury should be shorted relative to price, and the current WTI oil futures contract should be longed—all in the near term.

6. CONCLUSION

In this paper we provided a snapshot about how to graph volatility in the three-dimensional space, in order to interpret the price dynamics of equity, bond, and oil markets.

It has been possible through the use of the bi-harmonic spline interpolation as provided by MATLAB, which allowed us to describe such dynamics through the use of drift and volatility as in the option pricing framework. As result, this interpolation tool allows for a smooth transition between points, thus we can graph potential regions of drift and volatility for near future time periods.

A key element of this framework is a glimpse about the use of this approach to understand how volatility moves throughout the near term. Surface windows allow for the analysis of volatility from different time perspectives. In this way, a trader or a researcher may be able to understand support and resistance levels for a particular asset. Another tool used beside the surface, is the PDF curve for trading which can help to determine if current volatility is below or above the surface window average, even though a better approach is to analyze the CDF curve and determine the cumulative probability of different volatility ranges.

Further research in volatility surface should be in establishing how different GARCH methods can improve on volatility prediction. Another area of study could be in the development of methods to improve volatility spiking, such as probabilities between certain regimes switches.

References

Black, F., & Scholes, M. (1973). The valuation of options and corporate liabilities. *Journal of Political Economy*, *81*(1), 637–654.

Cottrell, P. (2015). *Dynamically hedging oil and currency futures using receding horizontal control and stochastic programming* (Doctoral dissertation).

Curve Fitting Toolbox. (n.d.). Retrieved April 5, 2015, from http://www.mathworks.com/products/curvefitting/features.html

Mandelbrot, B. B., & Hudson, R. L. (2004). *The (mis)behavior of markets: A fractal view of risk, ruin, and reward*. New York, NY: Basic Books.

Shiller, R. J. (2005). *Irrational exuberance.* Princeton, NJ: Princeton University Press.

Soros, G. (2003). *The alchemy of finance*. Hoboken, NJ: John Wiley & Sons, Inc.

Thaler, R. H. (1993). *Advances in behavioral finance*. New York: Russell Sage Foundation.

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